Proper Generalized Decomposition method to solve Quasi Static Field Problems

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Abstract— In the domain of numerical computation, the Proper Generalized Decomposition (PGD) method based on a finite sum of separable functions are now often used in mechanics and have shown their efficiency. In the paper, we propose to investigate the application of the PGD method to solve quasi static field problems with the A* potential formulation.

I. INTRODUCTION

Quasi static field problems are now mainly solved using the Finite Element Method. In the time domain, the magnetic field distribution has to be calculated for each time step. Moreover, at a given time step, this distribution depends on the magnetic field distribution calculated at the previous time steps. To solve this kind of problem, the Proper Generalized Decomposition method based on the separated representation of the solution in function of time and space can also be used [1][2]. The solution is then approximated by a finite sum of products of a function of space $R_n(x)$ defined on the domain D and a function of time $S_n(t)$ defined on the time interval I. The functions $R_n(x)$ and $S_n(t)$ are discretised using a Finite Element Approximation on D and I respectively. During the last years, the use of these techniques has strongly increased in the field of mechanic computation. In computational electromagnetism, the PGD method has been applied to study a fuel cell polymeric membrane model [3] or the skin effect in a rectangular slot using a 1D model [4].

In the paper, we propose to investigate the solution of a 2D quasi static field using the PGD. First, the quasi static field problem and the A* formulation are introduced. Then, the solution using the PGD approach is presented. Finally, the numerical model is applied in order to solve a 2D example using a Finite Element approximation.

II. MAGNETODYNAMIC FORMULATION

A. *Maxwell's equations*

Let's consider a domain D of boundary Γ ($\Gamma=\Gamma_B\cup\Gamma_H$ and $\Gamma_B\cap\Gamma_H=0$). In D, a conducting domain D_c of boundary Γ_c ($\Gamma=\Gamma_{Jind}\cup\Gamma_E$ and $\Gamma_{Jind}\cap\Gamma_E=0$) is introduced (Fig. 1).

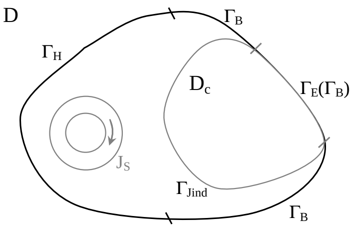

Fig.1. Magnetodynamic problem

The quasi static field problem can be described by the Maxwell's equations and the behaviour laws,

$$\mathbf{curl\,E} = -\partial_t\mathbf{B} \quad \text{(a)} \qquad \mathbf{curl\,H} = \mathbf{J}_{ind} + \mathbf{J}_s \quad \text{(b)} \qquad (1)$$

$$\text{div}\,\mathbf{B} = 0 \quad \text{(a)} \qquad \mathbf{div}\,(\mathbf{J}_{ind} + \mathbf{J}_s) = 0 \quad \text{(b)} \qquad (2)$$

$$\mathbf{B} = \mu_0\,\mu_r\,\mathbf{H} \quad \text{(a)} \qquad \mathbf{J}_{ind} = \sigma\,\mathbf{E} \quad \text{(b)} \qquad (3)$$

with μ_0 the magnetic permeability of the vacuum, μ_r the relative permeability of the material, σ the electrical conductivity, **B** the magnetic flux density, **H** the magnetic field, **E** the electric field, $\mathbf{J}_s$ the known current density flowing through the inductor and $\mathbf{J}_{ind}$ the eddy current density in the conducting domain. To impose the unicity of the solution, boundaries conditions must be added such as,

$$\mathbf{B.n}=0 \text{ on } \Gamma_B \text{ and } \mathbf{H}\times\mathbf{n}=0 \text{ on } \Gamma_H \qquad (4)$$

$$\mathbf{J}_{ind}.\mathbf{n}=0 \text{ on } \Gamma_J \text{ and } \mathbf{E}\times\mathbf{n}=0 \text{ on } \Gamma_E \qquad (5)$$

with **n** the outward unit normal vector, $\Gamma_B\cup\Gamma_H=\Gamma$ and $\Gamma_E\cup\Gamma_J=\Gamma_c$.

B. *A* Formulation*

To solve the previous problem, the A* formulation can be used. A modified magnetic vector potential A* is introduced in the whole domain by using (1-a) and (2-a), this can be expressed such that:

$$\mathbf{B} = \mathbf{curl\,A^*} \text{ and } \mathbf{E} = -\partial_t\mathbf{A^*} \text{ with } \mathbf{A^*}\times\mathbf{n}=0 \text{ on } \Gamma_B \qquad (6)$$

The relation (1-b) is solved in D×[0,T] with T the duration of the simulation. The weak form of (1-b) to be solved can be written such that:

$$\int_D\int_0^T \frac{1}{\mu}\mathbf{curl\,A^*}.\mathbf{curl\,A'} + \sigma\,\partial_t\mathbf{A}.\mathbf{A'}\,dtdD = \int_D\int_0^T \mathbf{J}_s.\mathbf{A'}\,dtdD \qquad (7)$$

with **A'** a test function defined in the same space of **A**.

III. PROPER GENERALIZED DECOMPOSITION

In order to solve the relation (7), a method based on the PGD approach can be used [2]. Then, the magnetic vector potential is approximated by a separated representation of space and time functions such that:

$$\mathbf{A^*}(\mathbf{x},t) = \sum_{n=1}^{M} R_n(\mathbf{x})\cdot S_n(t) \qquad (8)$$

with $\mathbf{x}\in D$, $t\in[0,T]$ and M the number of modes. To compute the functions $R_n(\mathbf{x})$ and $S_n(t)$, an iterative enrichment method is used. The couple $(R_n(x),S_n(t))$ is calculated regarding the previous couples $(R_i(x),S_i(t))$ with $i\in[1,n-1]$. Each couple $(R_n(x),S_n(t))$ is calculated by solving iteratively two equations determined from (7). These relations are deduced by applying separable test functions under the form $R^*_n(x)S^*_n(t)$.

To approximate the functions $R_n(\mathbf{x})$ and $S_n(t)$, a discretization in the spatial and time spaces is used. Then, the functions $R_n(\mathbf{x})$ can be approximated in 2D in the nodal shape function and $S_n(t)$ in a 1D node element space. Then, theses functions can be written under the form:

$$R_n(\mathbf{x}) = \sum_{i=1}^{Nn} R_{n,i} \cdot w^n{}_i(\mathbf{x}) \quad \text{and} \quad S_n(t) = \sum_{i=1}^{Nt} S_{n,i} \cdot w_i(t) \qquad (9)$$

with N_n the number of nodes of the spatial mesh, N_t the node number on the time interval, $R_{n,i}$ and $w^n{}_i(\mathbf{x})$ the value of $R_n(\mathbf{x})$ at the node i and the associated interpolation function and $S_{n,i}$ and $w_i(t)$ the value of $S_n(t)$ on the node i and its interpolation function.

IV. Application

A conducting plate submitted to a magnetic field created by a stranded inductor is considered in 2D (Fig. 2). This inductor is supplied by a sinusoidal current with a magnitude equal to 1A and a frequency of 200Hz. The 2D spatial mesh has 1330 nodes and 2560 triangles. The time interval of simulation is fixed to [0;0.0167s] and is discretised using 100 1D element. The problem has been solved using the PGD method. In Fig. 3, we present the evolutions of the time functions $S_n(t)$ for the four first modes. A transient state can be observed on all curves. In order to study the influence of each mode, we present in Fig. 4 and 5, the distribution of **curl**(R_1(x)) and **curl**(R_2(x)) can be interpreted as the distribution of the magnetic flux density **B** for the first and the second modes respectively. For the first mode, it seems that the distribution of **curl**(R_1(x)) does not take into account the effect to the conducting plate. For the second mode, the distribution of **curl**(R_2(x)) represents the reaction magnetic field due to the eddy current density. For the global solution associated with the sum of modes, the distribution of **B** is correct. The magnetic energy in the whole domain versus the time is given in Fig. 6. A transient state can be observed taken into account by the functions $S_n(t)$ (Fig. 3).

V. Conclusion

A Proper Generalized Decomposition method associated with the A* potential formulation has been developed in order to solve a 2D quasi static field problem. In the studied example, the solution obtained with the PGD converges quickly towards the solution.

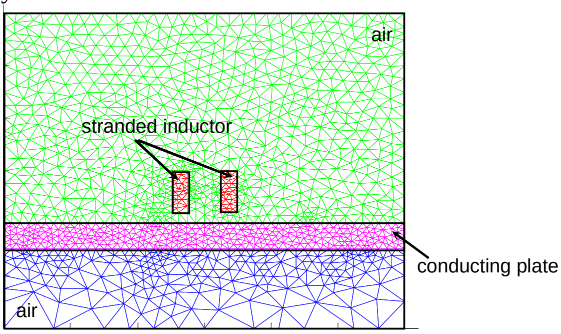

Fig.2. Description of the device and the 2D mesh

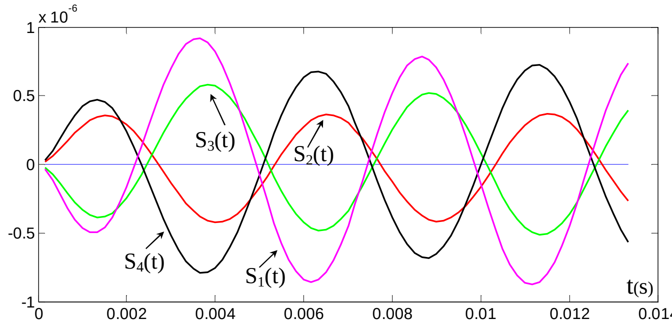

Fig.3. Results of the functions $S_n(t)$

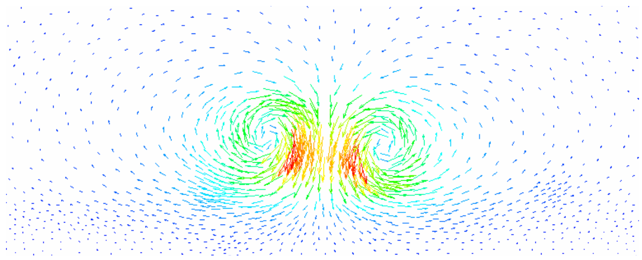

Fig.4. Distribution of **B** for the first mode (B_{max}=1.0mT)

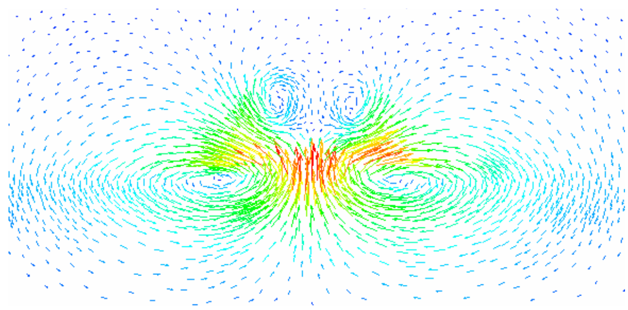

Fig.5. Distribution of **B** for the second mode (B_{max}=0.101mT)

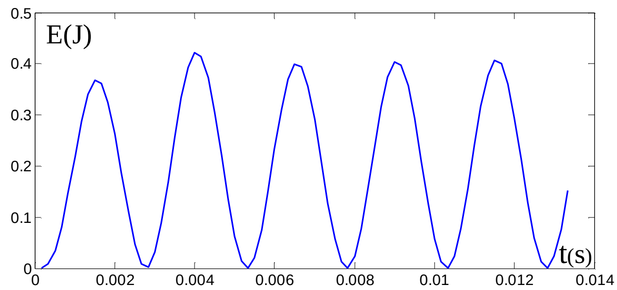

Fig.6. Magnetic energy versus the time

VI. References

[1] F. Chinesta, A. Ammar, E. Cueto, “Recent Advances and New Challenges in the Use of the Proper Generalized Decomposition for Solving Multidimensional Models”, Archives of Computational Methods in Engineering, Vol. 17, N° 4, pp. 327-350.

[2] A. Nouy, “A priori model reduction through Proper Generalized Decomposition for solving time-dependent partial differential equations”, *Computer Methods in Applied Mechanics and Engineering* Vol. 199, N° 23-24, pp.1603-1626.

[3] P. Alotto, M. Guarnieri,F. Moro, A. Stella “A proper generalized decomposition approach for modeling fuel cell polymeric membranes” CEFC2010, Chicago (USA).

[4] M. Pineda-Sanchez et all., Skin Effect in a Rectangular Slot via the Method of Separation of Variables, ISEF09, Arras(France).